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BUCKLING TESTS WITH A SPAR-RIB GRILL*

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SUMMARY

The present report deals with a comparison of mathematically and experimentally defined buckling loads of a spar-rib grill, on the assumption of constant spar section, and infinitely closely spaced ribs with rigidity symetrical to the grill center. The loads are applied as equal bending moments at both spar ends, as compression in the line connecting the joints, and in the spar center line as the assumedly uniformly distributed spar weight. The evaluation formula is valid for a deflection following a half wave, and confirmed by the tests within a scope sufficient for practical purposes. The formula is applicable with safety in the ratio bounded by by experiments between the end moments and the compressive force, or between the coefficients \underline{s} , \underline{m} , and \underline{m} , respectively.

An effect of the finite aspect ratio of test spars (8:100) is practically not noticeable. For smaller compressive forces and greater end moments the coefficients of the evaluation formula should be computed again for further deflection forms, in order to find further buckling conditions, which, of course, yield lower loads at the stability limit. For small spar weight a buckling condition exact for vanishing weight, which is written down for arbitrary wave number of the deflections, may be employed for the first comparison, which wave number gives the lowest buckling load. An extension of the energy equation and consequently the achievement of a buckling condition for any degree of rib rigidity along the rib axis and other load distribution is readily accomplished.

^{*}Uber Kippversuche mit einem Holm-Rippenrost. Luftfahrtforschung, vol. 17, no. 3, March 20, 1940, pp. 76-81.

Notation

- M1, M2* bending moment in the spar ends, mkg
- S₁, S₂ compressive force in line connecting the joints, kg
- G₁, G₂ spar weight, acting in beam axis, kg
- A1, A2 rigidity of transverse bending, kgm2
- C1, C2 torsional rigidity of spars, kgm2
 - Ar sum of rigidity of all ribs for buckling in the plane of the grill, kgm²
 - Br sum of rigidity of all ribs for buckling at right angle to plane of grill, kgm²
 - AAr strain energy of ribs for the spar length 1 by buckling in the plane of grill, kgm/m
 - BBr strain energy of ribs for the spar length 1 by buckling at right angle to plane of grill, kgm/m
- K1, K2, K3 nondimensional free value
 - h half distance of bean axes, m
 - distance of spar joints (spar length), m
- m1, m2 end moments of a rib for spar length 1, kgn/n
 - n half-wave number in the course of η , β_1 , and β_2
- c₁, c₂ numerical values of part integrals in the energy equation
 - x longitudinal coordinate, counted from a joint, n
 - y lateral deflection of beam axes at distance x from joint, m

^{*}Subscripts 1 and 2 denote - if used alone - that the particular quantity belongs to spar 1 and spar 2, respective-ly. Without subscript the quantities apply to both spars unless stated otherwise.

β₁, β₂ buckling angle (angle of rotation of spar sections at right angle to plane of grill)

α1, α2 influence figures

 $\xi = x/1$ nondimensional longitudinal coordinate

 $\eta = y/1 \sqrt{\frac{A_1 + A_2}{C_1 + C_2}}$ nondimensional lateral deflection

Nondimensional quantities in the energy equation:

$$\underline{\mathbf{m}}_{1} = \frac{2 \, \mathbf{M}_{1} \, \mathbf{1}}{\sqrt{(\mathbf{A}_{1} + \mathbf{A}_{2})(\mathbf{C}_{1} + \mathbf{C}_{2})}}, \quad \underline{\mathbf{m}}_{2} = \frac{2 \, \mathbf{M}_{2} \, \mathbf{1}}{\sqrt{(\mathbf{A}_{1} + \mathbf{A}_{2})(\mathbf{C}_{1} + \mathbf{C}_{2})}}$$

$$\underline{\mathbf{s}} = \frac{(\mathbf{S}_1 + \mathbf{S}_2) \mathbf{l}^2}{\mathbf{A}_1 + \mathbf{A}_2}$$

$$\underline{g_1} = \frac{G_1 l^2}{\sqrt{(A_1 + A_2)(C_1 + C_2)}}, \quad \underline{g_2} = \frac{G_2 l^2}{\sqrt{(A_1 + A_2)(C_1 + C_2)}}$$

$$\underline{a} = \frac{2 \text{ Ar 1}}{(A_1 + A_2)h}, \quad \underline{b} = \frac{2 \text{ Br 1}}{(C_1 + C_2)h}$$

$$\mu_1 = \frac{2 C_1}{C_1 + C_2}, \quad \mu_2 = \frac{2 C_2}{C_1 + C_2}$$

$$\zeta_1 = \frac{\alpha_1}{\alpha_1^2 - \alpha_2^2}, \quad \zeta_2 = \frac{\alpha_2}{\alpha_1^2 - \alpha_2^2}, \quad \zeta_3 = \frac{1}{\alpha_3 - \alpha_4}$$

INTRODUCTION

Earlier reports give buckling loads which were derived under specified simplifying premises and assumptions (references 1 and 2). In particular, the spars were assumed as small rails with vanishing thickness and equal constant section for both spars. It is further assumed that the rib rigidity is constant and uniformly distributable across the length of the spar. (Infinitely closely spaced ribs as substitute of finite number of ribs.) End bending moments and end compressive forces were assumed as spar loads.

The following describes an investigation which enables a comparison of computed and experimentally defined buckling loads of a spar-rib grill. The experimental grill has dissimilar spars which are not "very small" (8/100 mm), the number of ribs, 8, is comparatively low). The rigidity of the ribs is no longer constant across the axis, although not symmetrically distributed with respect to the rib center. In the evaluation formula, which ordinarily is derived under the same general assumptions from the energy equation as before (reference 2), the dissimilarity of the spars, the cited course of rib rigidity, and the spar weight are allowed for.

Experimental Arrangement*

The spars of the model grill are two pine boards 8 by 100 by 3250 mm, selected for grain, absence of knots, etc., and the ribs (7 by 7 by 1000 mm) of the same grade of pine with reinforcement strips glued at the ends (fig. 2). Spar and rib end are fastened together by a 12-by-12by-0.5-mm hard drawn brass angle and five 3-mm bolts. The spacing of the spar axes from each other amounts to 2h = 1025 mm. A hardened steel cylinder drilled out with a taper from each side and fitting tightly in the spar end reinforced by plywood shims, serves as nodal point of the spar ends. The annular edge within the cylinder has an 8-mm diameter and rests against a hardened round steel bar of 7-mm diameter. This in turn is fastened on one grill end to the support, on the other spar end the slidability, shown in figures 1 and 2, parallel with the line connecting the nodal points, is provided. The end moments are transmitted over fixed end levers built up of 10-by-1-mm flat brass and 10-by-10-by-1-mm angle brass, which project to 560-mm load application. For applying the axial compression, the fixed joint near the spars carries two ballbearing pulleys over which the tension of the loading, weights is carried parallel with the connecting line of the joints to the movable joint. Wooden bars with metal sting fitted at the end levers of spar 2 record the tangential inclination of the elastic line in the joints, on a little board with a sheet of paper. The length of the tracing levers from joint to metal sting is 1020 mm. The end moment existing on spar las a result of the weight of

^{*}Sec fig. 1.

the end levers and of the overhanging spar portion is 0.082 mkg, and 0.100 mkg on spar 2 because of the tracing levers. The effective weight on the joints (weight of spars plus weight of ribs) is equalized for each spar at 1.21 kg. The buckling angles in joints are preserved by 1-m wooden levers running parallel upward from the spar ends with a loose pin or stud at the end parallel to the connecting line of the joints. The guide is adjustable perpendicularly to the line connecting the joints. ure 4 shows the first set-up, where the spar plus the rib weight was upwardly compensated by pulleys. But this compensation had to be abandoned in favor of a more complicated evaluation formula because the oscillations of the grill could not be removed quickly enough after load changes. Neither figure 4 nor figure 3 represents the final method.*

Evaluation Formula

The energy equation for elastically identical spars with a load through M_1 , M_2 and S_1 and S_2 and ribs of constant section (reference 2) reads:

$$J = \int_{0}^{2} \left\{ (M_{1}\beta_{1} + M_{2} \beta_{2})y'' - \left(\frac{S_{1} + S_{2}}{2} - 3\frac{Ar}{1h}\right)y'^{2} + \frac{Br}{1h} (\beta_{1}^{2} + \beta_{1}\beta_{2} + \beta_{2}^{2}) + \frac{1}{2}C(\beta_{1}^{12} + \beta_{2}^{12}) + Ay''^{2} \right\} dx$$
(1)

or in nondimensional coordinates and coefficients

$$\underline{F} = \int_{0}^{1} \left\{ \underline{m}_{1} \beta_{1} + \underline{m}_{2} \beta_{2} \right\} \eta^{"} - (\underline{s} - 3 \underline{a}) \eta^{!2} + \underline{b} (\beta_{1}^{2} + \beta_{1} \beta_{2} + \beta_{2})^{2} + \frac{1}{2} (\beta_{1}^{12} + \beta_{2}^{12}) + \eta^{"2} \right\} d\xi$$
(2)

M1 and M2 are the principal moments bending at point x.

^{*}The tests were made in the German Technical Institute at Brünn, but had to be interrupted during the general mobilization in September 1938. The means for the tests had been furnished by the National Research Council of Czechoslovakia.

Accordingly, by virtue of the annexed loading schedule, M_1 and M_2 must be replaced by, respectively,

$$M_1 - \frac{G_1}{21}(xl - x^2)$$
 and $M_2 - \frac{G_2}{21}(xl - x^2)$

For the derivation of the proportions with Ar and Br the buckling of a rib at right angle to the plane of the grill due to the reaction parts transmitted at the joints, is expressed conformally to figure 7, with

$$\beta_1 = \frac{hl}{Br}(m_1 \alpha_1 - m_2\alpha_2), \quad \beta_2 = \frac{hl}{Br}(m_2\alpha_1 - m_1 \alpha_2) \quad (3)$$

(The rib rigidity is $\frac{Br}{l}$, the rigidity for spar length 1.) Then the energy content of the rib is

$$A_{Br} = \frac{Br}{2 h l(\alpha_1^2 - \alpha_2^2)} (\alpha_1(\beta_1^2 + \beta_2^2) + 2 \alpha_2 \beta_1 \beta_2)$$
 (4)

For buckling in the plane of the grill, it affords in the same way

$$A_{Ar} = \frac{Ar}{h l(\alpha_3 - \alpha_4)} y!2$$
 (5)

if y' (reference 2) replaces β_1 and β_2 , and α_3 and α_4 replace α_1 and α_2 .

The sum then is

$$J = \int_{0}^{1} \left\{ \left(M_{1} - \frac{G_{1}}{21} (x 1 - x^{2}) \right) \beta_{1} + \left(M_{2} - \frac{G_{2}}{21} (x 1 - x^{2}) \right) \beta_{2} \right\} y''$$

$$- \left(\frac{S_{1} + S_{2}}{2} - \frac{Ar}{h1(\alpha_{3} - \alpha_{4})} \right) y'^{2} + \frac{Br(\alpha_{1}(\beta_{1}^{2} + \beta_{2}^{2}) + 2\alpha_{2}\beta_{1}\beta_{2}}{2 h 1(\alpha_{1}^{2} - \alpha_{2}^{2})}$$

$$+ \frac{1}{2} C_1 \beta_1^{12} + \frac{1}{2} C_2 \beta_2^{12} + \frac{1}{2} (\Lambda_1 + \Lambda_2) y^{12} \bigg] d x \qquad (6)$$

Extension with $\frac{21^2}{C_1 + C_2}$, followed by introduction of the nondimensional coordinates and abbreviations, gives:

$$\underline{F} = \int_{0}^{1} \left[\left\{ (\underline{m}_{1} - \underline{g}_{1}(\xi - \xi^{2}))\beta_{1} + (\underline{m}_{2} - \underline{g}_{2}(\xi - \xi^{2}))\beta^{2} \right\} \eta^{\parallel} - (\underline{g} - \xi_{3} \underline{a})\eta^{2} + \frac{1}{2} \underline{b} (\xi_{1}(\beta_{1}^{2} + \beta_{2}^{2}) + 2 \xi_{2} \beta_{1}\beta_{2}) + \frac{1}{2} \mu_{1}\beta_{1}^{2} + \frac{1}{2} \mu_{2}\beta_{2}^{2} + \eta^{\parallel 2} \right] d\xi$$
(7)

The limiting conditions adapted to the described test arrangement read (case 1, reference 2):

$$\eta(0) = \eta(1) = \eta''(0) = \eta''(1) = 0
\beta_1(0) = \beta_1(1) = \beta_2(0) = \beta_2(1) = 0$$
(8)

These are experimentally much easier to keep than others and give at the same time lower buckling loads in ratio to built-in spars.

The related differential equations following from the disappearance of the first variation of equation (7) are:

$$\frac{1}{2} \frac{d^{2}}{d \xi^{2}} \left\{ (\underline{m}_{1} - \underline{g}_{1}(\xi - \xi^{2})) \beta_{1} + (\underline{m}_{2} - \underline{g}_{2}(\xi - \xi^{2})) \beta_{2} \right\} + \eta^{IV} + (\underline{s} - \xi_{3}\underline{a}) \eta^{"} = 0$$
(9)

$$\frac{(\underline{m}_{1} - \underline{g}_{1} (\xi - \xi^{2}))\eta'' - \mu_{1}\beta_{1}'' + \underline{b}(\xi_{1}\beta_{1} + \xi_{2}\beta_{2}) = 0}{(\underline{m}_{2} - \underline{g}_{2}((\xi - \xi^{2}))\eta'' - \mu_{2}\beta_{2}'' + \underline{b}(\xi_{2}\beta_{1} + \xi_{1}\beta_{2}) = 0}$$
(10)

After two integrations of equation (9) the integration constants disappear as a result of the chosen limiting conditions, leaving the simpler relation

$$\frac{1}{2} \left(\underline{\mathbf{m}}_{1} - \underline{\mathbf{g}}_{1} (\xi - \xi^{2}) \right) \beta_{1} + \frac{1}{2} \left(\underline{\mathbf{m}}_{2} - \underline{\mathbf{g}}_{2} (\xi - \xi^{2}) \right) \beta_{2} + \eta'' + \left(\underline{\mathbf{s}} - \xi_{3} \underline{\mathbf{a}} \right) \eta = 0 \quad (9a)$$

For vanishing spar weight equations (9) and (10) are satisfied by

$$B_1 = \kappa_1 \sin n \pi \xi$$
, $\beta_2 = \kappa_2 \sin n \pi \xi$, $\eta = \kappa_3 \sin n \pi \xi$ (11)

The final determinant remaining after the insertion of the solutions (11) gives the buckling condition

$$\frac{1}{2} \frac{m_1^2}{(n\pi)^2} (\mu_2(n\pi)^2 + \zeta_1 \underline{b}) + \frac{1}{2} \frac{m_2^2}{(n\pi)^2} (\mu_1(n\pi)^2 + \zeta_1 \underline{b})
- \frac{m_1 \underline{m}_2}{(n\pi)^2} \zeta_2 \underline{b} + (\frac{\underline{s} - \zeta_3 \underline{a}}{(n\pi)^2} - 1)
\{\mu_1 \mu_2(n\pi)^2 + 2 \zeta_1 \underline{b} + (\zeta_1^2 - \zeta_2^2) \underline{b}^2 / (n\pi)^2\} = 0$$
(12)

The exact solution of the complete differential equations was foregone for reasons of the paper work involved, and the evaluation formula therefore derived by presentation of suitably chosen approximating theorems in the energy equation. Chosen as such were:

I.
$$\beta_1 = \frac{1}{\kappa_1} (\xi - \xi^2), \ \beta_2 = \frac{1}{\kappa_2} (\xi - \xi^2), \ \eta'' = \frac{1}{\kappa_3} (\xi - \xi^2)$$
 (13)

II.
$$\beta_1 = \frac{1}{\kappa_1} (\xi - 2\xi^3 + \xi^4)$$
, $\beta_2 = \frac{1}{\kappa_2} (\xi - 2\xi^3 + \xi^4)$, $\eta'' = \frac{1}{\kappa_3} (\xi - 2\xi^3 + \xi^4)$ (14)

In order to clarify the mutual relations of the coefficients in the buckling equation the numerical values of the part integrals in the energy equation are generalized:

$$\frac{1}{\kappa_{1}^{2}} \int_{0}^{1} \beta_{1}^{2} d\xi = \frac{1}{\kappa_{2}^{2}} \int_{0}^{1} \beta_{2}^{2} d\xi = \frac{1}{\kappa_{1}\kappa_{2}} \int_{0}^{1} \beta_{1}\beta_{2} d\xi$$

$$= \frac{1}{\kappa_{3}^{2}} \int_{0}^{1} \eta^{*} d\xi = \frac{1}{\kappa_{1}\kappa_{3}} \int_{0}^{1} \beta_{1}\eta^{*} d\xi = \frac{1}{\kappa_{2}\kappa_{3}} \int_{0}^{1} \beta_{2}\eta^{*} d\xi = c_{1}$$
(15)

$$\frac{1}{\kappa_{1}^{2}} \int_{0}^{1} \beta_{1}^{1} e^{2} d\xi = \frac{1}{\kappa_{2}^{2}} \int_{0}^{1} \beta_{2}^{1} d\xi = c_{2}$$
 (16)

$$\frac{1}{\kappa_3^2} \int_0^1 \eta^{12} d\xi = c_3$$
 (17)

Total M. Teather C. J.

$$\frac{1}{\kappa_{1}\kappa_{3}} \int_{0}^{1} \eta'' \beta_{1}(\xi - \xi^{2}) d\xi = \frac{1}{\kappa_{2}\kappa_{3}} \int_{0}^{1} \eta'' \beta_{2}(\xi - \xi^{2}) d\xi = c_{4} \quad (18)$$

With these data the energy equation (7) supplies as buckling condition

$$\frac{1}{2} \left(\mu_{2} \frac{c_{2}}{c_{1}} + \zeta_{1} \underline{b} \right) \left[\frac{c_{1}}{c_{2}} m_{1}^{2} - 2 \frac{c_{4}}{c_{2}} \underline{m}_{1} \underline{g}_{1} + \underline{g}_{1}^{2} \frac{c_{4}^{2}}{c_{1} c_{2}} \right] \\
+ \frac{1}{2} \left(\mu_{1} \frac{c_{2}}{c_{1}} + \zeta_{1} \underline{b} \right) \left[\frac{c_{1}}{c_{2}} \underline{m}_{2}^{2} - 2 \frac{c_{4}}{c_{2}} \underline{m}_{1} \underline{g}_{1} + \underline{g}_{2}^{2} \frac{c_{4}^{2}}{c_{1} c_{2}} \right] \\
- \underline{b} \zeta_{2} \left[\frac{c_{1}}{c_{2}} \underline{m}_{1} \underline{m}_{2} - \frac{c_{4}}{c_{2}} (\underline{m}_{1} \underline{g}_{1} + \underline{m}_{2} \underline{g}_{2}) + \underline{g}_{1} \underline{g}_{2} \frac{c_{4}^{2}}{c_{1} c_{2}} \right] \\
+ \frac{c_{3}}{c_{1}} (\underline{s} - \zeta_{3} \underline{a}) - 1 \right) \left\{ \mu_{1} \mu_{2} \frac{c_{2}}{c_{1}} + 2 \zeta_{1} \underline{b} + (\zeta_{1}^{2} - \zeta_{2}^{2}) \underline{b}^{2} \frac{c_{1}}{c_{2}} \right\} = 0$$
(19)

The numerical values of c₁, c₂... and their combinations are given for approximations I and II in the second and third lines of table I.

A comparison of equation (19) with the exact solution (12) discloses, then, $\frac{c_2}{c_1}$ and $\frac{c_1}{c_3}$ for π^2 . Moreover, the limiting conditions considered here give for Timoshenko's (reference 3) exact buckling factor

$$G = 28.3$$
 (20)

for $\frac{c_1c_2}{c_4^2}$ the actual value $\frac{1}{4}$ 28.3° = 200.223. The third approximation and final evaluation formula employed is that in which $\frac{c_2}{c_1}$, $\frac{c_1}{c_3}$, and $\frac{c_1c_2}{c_4^2}$ are replaced by the exact values and $\frac{c_2}{c_4}$ by the value 44.4535, the latter following from $\frac{c_2}{c_1} = \pi^2$ and $\frac{c_1c_2}{c_4^2} = 200.223$ and which probably differs very little and nonessentially from the true value

$$\frac{1}{2} \left(\mu_{2} \pi^{2} + \zeta_{1} \underline{b} \right) \left[\frac{\underline{m}_{1}^{2}}{\pi^{2}} - \frac{\underline{m}_{1}}{22.23} + \frac{\underline{g}_{1}^{2}}{200.22} \right] \\
+ \frac{1}{2} \left(\mu_{1} \pi^{2} + \zeta_{1} \underline{b} \right) \left[\frac{\underline{m}_{2}^{2}}{\pi^{2}} - \frac{\underline{m}_{2}}{22.23} + \frac{\underline{g}_{2}^{2}}{200.22} \right] \\
- \zeta_{2} \underline{b} \left[\frac{\underline{m}_{1} \underline{m}_{2}}{\pi^{2}} - \frac{\underline{m}_{1} \underline{g}_{1}}{44.45} + \frac{\underline{g}_{1} \underline{g}_{2}}{200.22} \right] \\
+ \left(\underline{\underline{s} - \zeta_{3} \underline{a}}{\pi^{2}} - 1 \right) \left\{ \mu_{1} \mu_{2} \pi^{2} + 2 \zeta_{1} \underline{b} + \left(\zeta_{1}^{2} - \zeta_{2}^{2} \right) \frac{\underline{b}^{2}}{\pi^{2}} \right\} = 0 \quad (21)$$

 A_1 , C_1 , A_2 , C_2 were defined from buckling tests with one single spar each without ribs attached. The related evaluation formula follows from the above after insertion of $M_1 = M_2 = M$, $S_1 = S_2 = S$, a = 0, b = 0, and the expressions for s, m_1 , and m_2 in the form

$$\frac{M^2 l^2}{\pi^2} - \frac{G l^2 M l}{44.45} + \frac{G^2 l^4}{800.89} + \frac{C S l^2}{\pi^2} = A C$$
 (22)

Test Procedure

The first tests were made without the weights suspended from the end lever, one horizontal pull cord being used to transmit the principal part of the load, and then the other. After various attempts the locations of the cylinders slipped in the spar ends were finally established for which the deflections by equal load near the stability limit were lowest. The inclination of the elastic line at the spar ends served, as already indicated, as deflection component, the board with the sheet of paper being pressed against the tracing lever after the set-up ceases to swing and the load is written alongside the tracing mark. The adjusting levers for end-buckling angle remained loosely in the guides. For further tests with this setting the grill was fitted with 7 kg end-lever loads on spar 1, on spars 1 and 2, and then on spar 2. The applied load at the end levers could be shifted transversely to the beam axis, and so far that after application of the endlever load the resetting lever over the joints was loose again in the guides. Thus the end load was applied in consistently smaller stages till the lateral deflection in

spar center reached about 100 mm. Greater deflections produced appreciable permanent deformations. Up to 100 mm lateral deflection in spar center the permanent deflection, after applied end load, was at the most 1.5 mm at the pointer tip. For the same reasons the recording of a supercritical branch of the stress-strain diagram, as by Prandtl (reference 4) was omitted. The height of the critical end load had to be determined, therefore, as hyperbolic asymptote height by known asymptote directions and three given hyperbola points (reference 5). The force deflection diagrams taken at both ends of spar 2 differ very little, which is indicative of the symmetrical course of the deflections. Tests with 10-kg endlever load left perceptible permanent deformations, especially in the spar-rib connections and the symmetrical course of the deflections disappeared. The evaluation formula would then give too high comparative values. Experiments and comparisons in this direction with an improved set-up are under way.

Table II contains the end moments M_1 , M_2 applied at the grill, and the related end compressive forces determined from the deflection plots. Figure 8 is the diagram showing the comparatively greater departure from the presumed symmetry of deflection, whose equilibrium curves have the relatively greater distance from the asymptotes. $S_1 + S_2 = 11.26$ kg is the sum of the "scale loading" of 10.8 kg determined by the asymptote construction and the "scale weight" of 0.457 kg identical for all the tests.

Spar Rigidity A1, C1, A2, C2

Following the buckling tests with the model grill the ribs were removed and each spar tested at three different end moments. The effective weight of each spar was 0.980 kg. The three load systems are appended in table III. Equation (22) gives for A_1 , C_1 , A_2 , C_2 three equations each which, interconnected, give each three pairs of rigidity A and C. Their averages

 $A_1 = 4.961 \text{ m}^2\text{kg}$ $C_1 = 0.605 \text{ m}^2\text{kg}$ $C_2 = 0.633 \text{ m}^2\text{kg}$

served as a basis in the subsequent calculations.

Determination of Rigidity Factors $\zeta_1 \underline{b}$, $\zeta_2 \underline{b}$, and $\zeta_3 \underline{a}$

Two each of the eight ribs spaced 15 cm apart were attached to a 20-cm long piece of spar in the same way as on the grill and supported and loaded. The torsion angles over the joints were determined by means of mirror reading and compiled in table IV. It shows the extent to which the symmetry assumption is satisfied for the course of rib rigidity along the rib axis. Equation (3) and the equations defining b, \(\frac{1}{2}\), and \(\frac{1}{2}\) give

$$\underline{b}\,\zeta_1 = \frac{2\,m_1\,l^2\beta_1}{(C_1 + C_2)(\beta_1^2 - \beta_2^2)} \quad \text{and} \quad \underline{b}\,\zeta_2 = \frac{2\,m_1\,l^2\beta_2}{(C_1 + C_2)(\beta_1^2 - \beta_2^2)} \tag{23}$$

and in our case for the mean values of β_1 , $\overline{\beta}_2$, and β_2 , $\overline{\beta}_1$:

$$\frac{1}{2} = 50.6, \quad \frac{1}{2} = 26.5$$

$$\frac{1}{2} |\beta_1| + \frac{1}{2} |\overline{\beta}_2| = 0.0423, \quad \frac{1}{2} |\beta_2| + \frac{1}{2} |\overline{\beta}_1| = 0.0218$$

Test 4 without end-lever load affords $\zeta_3 a$, which gives $S_1 + S_2 = 16.52$ kg. The evaluation formula (21) then gives

$$\zeta_3 \underline{a} = 5.14$$

According to the definition of $\zeta_{3\underline{a}}$, $\zeta_{1\underline{b}}$ and $\zeta_{2\underline{b}}$ should be - equal rigidity of ribs and their attachments in and at right angles to the plane of the grid being assumed -

$$\zeta_{3}\underline{a} = (\zeta_{1} + \zeta_{2})\underline{b} \frac{C_{1} + C_{2}}{A_{1} + A_{2}} = 77.1 \frac{1.238}{9.949} = 9.6$$

The much lower actual value of 5.14 is due to the fact that the rib fittings are much more elastic with respect to buckling in the plane of the grill than at right angles to the plane of the grill. It might be noted in passing that the eight ribs had been selected from about 80 others. The sequence of arrangement on the spar is that of table IV, hence the pair closest to the average value is situated at spar center where the greatest twist occurred.

Comparison of Theoretical and Experimental Values

Table II gives the end moments, the critical compressive forces found from the force-deflection diagrams, the mathematical compressive forces, and the difference between the compressive forces in percent. In view of the uncertainty affecting the determination of the critical compression forces from force-deflection diagrams and particularly because of the dissimilarity of the ribs, the accord between theory and test may be termed satisfactory. Since the evaluation formula for the small rail is exactly valid, the critical experimental compressive forces should, on the whole, exceed the theoretical, which is the case in test 1 only. For the present 8:100 aspect ratio a pure moment loading through M1 and M2 with and without allowance for finite aspect ratio would give a difference of about 1 percent, which, however, is covered by the uncertainty of the coordination. In test 3 the difference between the theoretical and the experimental critical compressive force with -6.2 percent is greatest compared with the two previous tests. This can be associated with the previously stressed maximum departure from the symmetry hypothesis for the deformation and this in turn with the dissimilarity of the ribs.

Translation by J. Vanier, National Advisory Committee for Aeronautics.

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TABLE I

Approx- imation	Cı	C2	c ₃	C4	C 2	C 2	C1 C2	C ₁
I	1 30	1 3	17 35×144	1 140	10	46.6	217.7	9.88235
II	31 18 x35	17 35	691 99×1400	37 3465	9.870968	45.486	209.607	9.86975.
III	-		-	_	π ²	44.4535	200.223	π²

TABLE II Critical Outside Loads of Grill

Test	Mı	Ma	(S ₁ +S ₂) _{test}	(S1+S2) theory	F*
	(mkg)	(mkg)	(kg)	(kg)	(percent)
1	0.082	4.02	12.29	11.95	2.8
2	4.002	4.02	11.56	11.82	-2.2
3	4.002	.10	11.26	12.01	-6.2
4	.082	.10	16.52	16.52	-

^{*}Difference between experimental and theoretical value in percent of computed value.

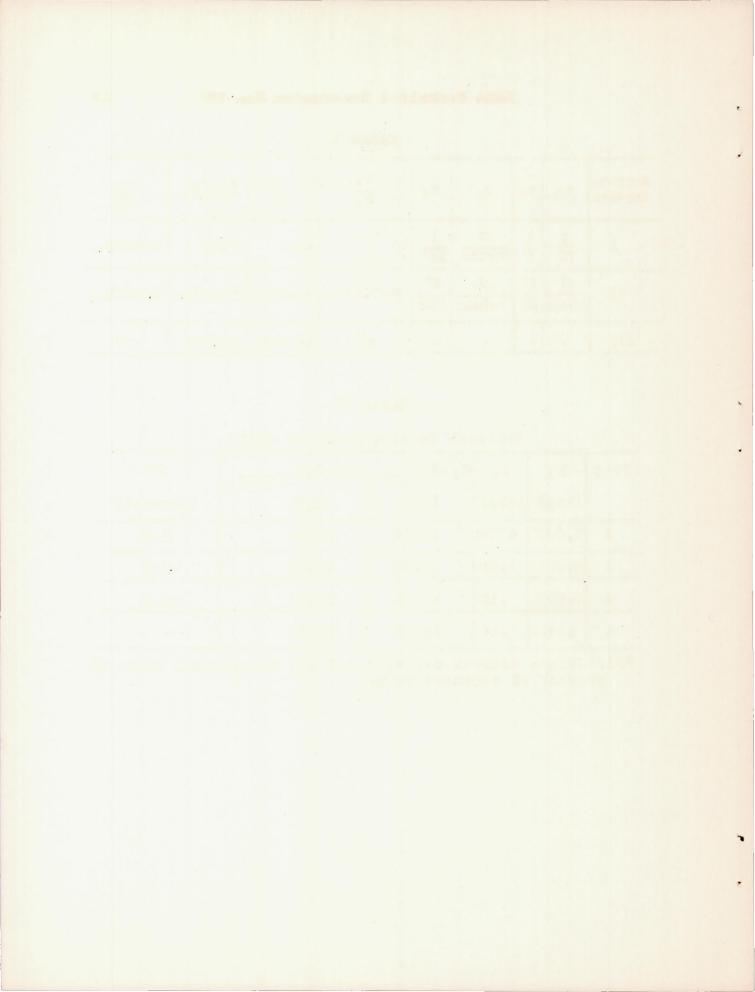


TABLE III Buckling Loads for the Free Spars

Spar	1	Spar 2		
M (mkg)	S (kg)	M (mkg)	s (kg)	
0.10	5.35	0.10	5.38	
1.21	4.14	1.21	4.22	
1.78	1.95	1.78	2.115	

TABLE IV

Rib Torsion Angle for $m_1 \frac{1}{4}$ and $m_2 \frac{1}{4} = 0.0813$ mkg							
Rib	βι	βε	β ₁	β ₂			
1 2	0.0435	-0.0225	0.0228	-0.0438			
3 4	.0431	0222	.0218	0427			
5 6	.0419	0214	.0215	0416			
7 8	.0409	0212	.0210	0409			
Average	.04235	021825	.021775	04225			

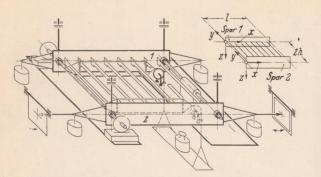


Figure 1 .- Experimental set up.

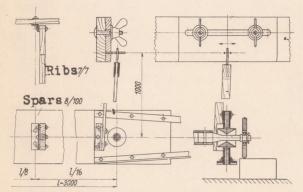


Figure 2.- Spar support, rib fitting and adjustment of end buckling angle.

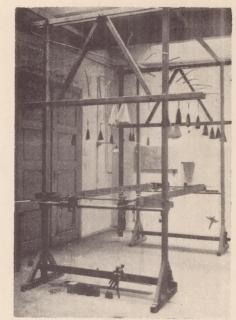


Figure 4.- First experimental set up.

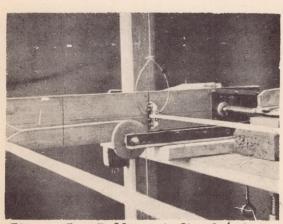


Figure 3.- Pulley at fixed joint.

$S_1 + S_2 - 9457 \text{ kg Scale weight.}$

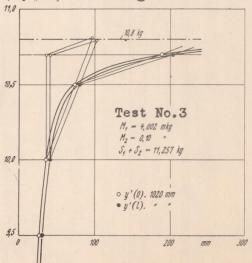
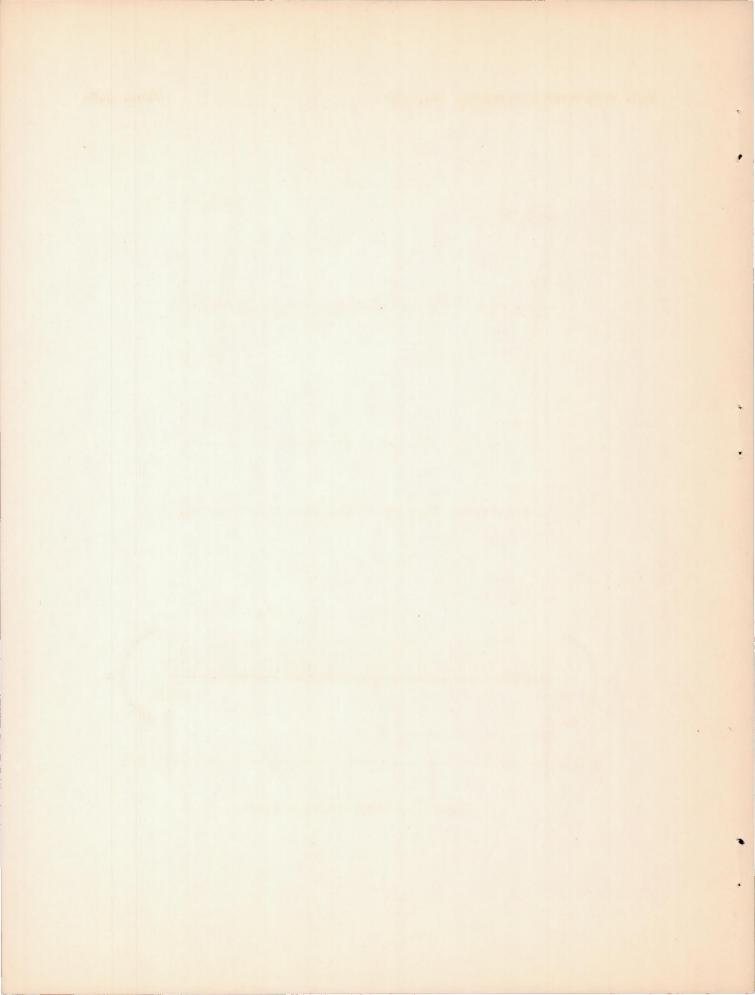


Figure 8.- Load-deflection diagram for test No. 3.



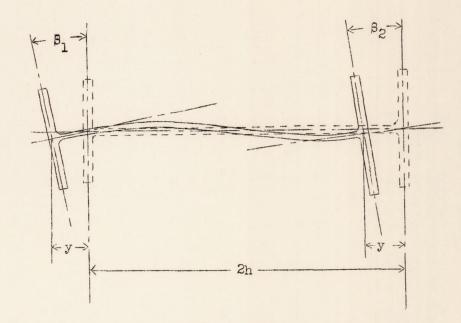


Figure 5.- Section at distance x from joint.

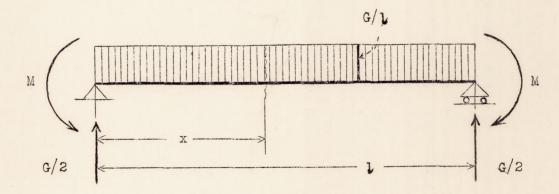
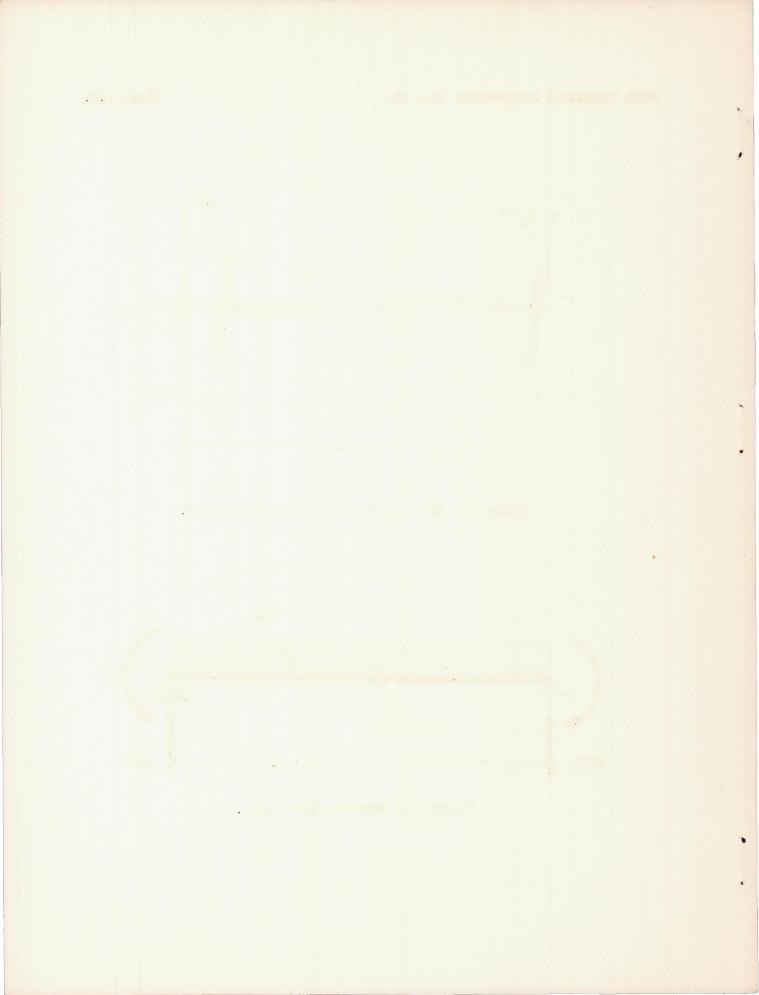


Figure 6.- Spar-bending loads.



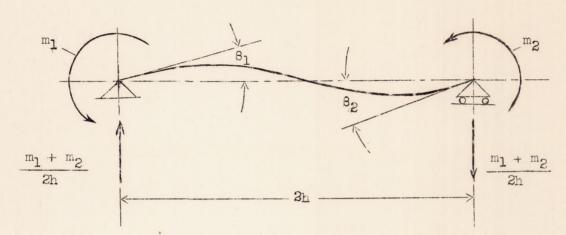


Figure 7.- Strain of ribs due to bending at right angle to plane of grill.

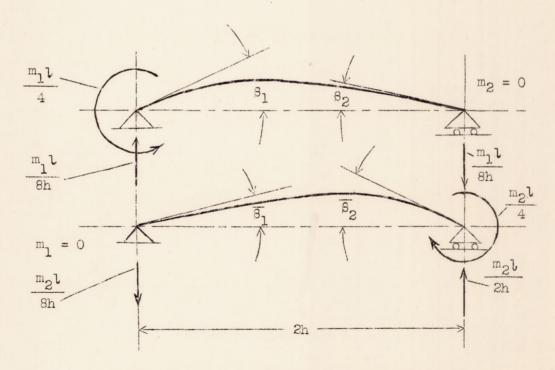


Figure 9.- Test rigging for determining Br .